

PROCEDURE **CinSi** (x: REAL): LibComplex.Complex

Returns the Cosine and Sine integrals as respectively the real and imaginary parts of the result.

The Cosine integral  $C_{in}$  is defined as:

$Cin(x) = \text{Integral } [0 .. x] \text{ of } (1 - \cos(t)) / t.$

$$Cin(x) = \int_{t=0}^x \frac{1 - \cos(t)}{t} dt$$

$$C_{in}(x) \equiv \int_{t=0}^x \frac{1 - \cos(t)}{t} dt$$

The Sine integral  $S_i$  is defined as:

$Si(x) = \text{Integral } [0 .. x] \text{ of } \sin(t) / t.$

$$Si(x) = \int_0^x \frac{\sin(t)}{t} dt$$

$$S_i(x) \equiv \int_0^x \frac{\sin(t)}{t} dt$$

Sample value :  $CinSi(1) = 0.23981\ 17420 + 0.94608\ 30703\ i.$

[Note that the Cosine integral  $Ci$  is defined differently, namely as  $Ci(x) = \text{Ln}(x) + \text{gamma} - Cin(x)$  where  $\text{gamma} = 0.5772156649015329\dots$ ]

The method employed is based on the method in Numerical Recipes, ie power series for small x, complex continued fraction for large x.

This function (ie  $Cin(x) + i * Si(x)$ ) is in fact the value of the exponential integral  $Ein(i * x)$ .

Accuracy is better than 13 decimal places.